tion due to the effect of polarization would not be detected in the present experiment.

CONCLUSION

The present paper presents experimental evidence for the existence of the theoretically predicted relativistic rise in the rate of ionization up to values of $p/\mu=100$. Our results are in disagreement with the results found by Goodman, Nicholson, and Rathgeber[14] using a proportional counter, but they are in good agreement with the results of Ghosh, Jones, and Wilson[15] who used a cloud chamber for their ionization measurement.

We wish to thank the staff and personnel of the Harvard Physics Department who made this work possible, and in particular we want to thank Professor J. C. Street and Dr. G. M. Nonnemaker for their interest and many valuable suggestions.

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The Stopping Cross Section of D_2O Ice*

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The energy loss of protons and deuterons in D_2O ice has been measured over the energy range $E_p=18-541$ kev using the double focusing magnetic spectrometer to measure the energy of the particles after they have traversed a known thickness of the ice target. One method of measurement is used to determine relative values of the stopping cross section as a function of energy, another method measures the absolute values. The results are in very good agreement with the values calculated from Bethe's semi-empirical formula. Possible sources of error are considered and the accuracy of these measurements is estimated to be ±4 percent.

I. INTRODUCTION

A KNOWLEDGE of the stopping cross section of the target material is essential in the measurements of nuclear cross sections if the target cannot be weighed to determine its thickness or if a thick target is used. In the course of making measurements of the $D(d,p)T$ cross section it seemed advisable to remeasure the stopping cross section for the D_2O ice targets, since the poor accuracy claimed for earlier measurements[1] would limit the accuracy with which the $D(d,p)T$ cross section could be measured. Further interest lay in the striking disagreement of the earlier measurements of the stopping cross section with the theoretical predictions. The double focusing magnetic spectrometer for charged particles is admirably suited to measurements of stopping cross sections, and this application of the instrument is described in this paper.

II. ABSOLUTE VALUE MEASUREMENTS

When a beam of protons is scattered from a thick target, the momentum spectrum of the scattered protons, as observed in the magnetic spectrometer, is made up of a succession of steps, one for each isotope in the target. Such a step spectrum, for protons scattered from O^{16} in D_2O, is shown by the solid curve in Fig. 1. The height of the step $N_{\max}$, is proportional to the scattering cross section $(d\sigma/d\omega)$ and inversely proportional to the effective stopping cross section ϵ_{eff} of the target material. $N_{\max}$ can be measured easily since the top of the step is nearly flat, and if $(d\sigma/d\omega)$ is known ϵ_{eff} can be determined from the following expression[2]

$$\begin{aligned}\epsilon_{\text{eff}}&\equiv\epsilon(E_1)(\partial E_2/\partial E_1)+\epsilon(E_2)(\cos\theta_1/\cos\theta_2)\\&=(d\sigma/d\omega)_{\theta_c}(E_2Q/N_{\max})(\Omega_c/R_c)4\pi\times10^{-15}\text{ ev-cm}^2,\end{aligned}$$

where $(d\sigma/d\omega)_{\theta_c}$ is the scattering cross section in millibarns per steradian at the angle θ_c in the center-of-mass system; Ω_c is the solid angle subtended by the aperture of the magnetic analyzer at the target (c.m. system); R_c is the momentum resolution, $P/\Delta P$, a known function of the geometry of the spectrometer. $N_{\max}/Q$ is the number of scattered particles of energy E_2 (measured in ev) counted in the spectrometer per Q microcoulombs of protons incident on the target, and $N_{\max}$ is to be evaluated for particles scattered from the front surface of the target. The molecular stopping cross section for protons of energy E is given by $\epsilon(E)=(1/N)dE/dX$, where N is the number of molecules per cubic centimeter and dE/dX is the energy loss per cm of path for protons of energy E. The subscripts 1 and 2 refer to incident and scattered particle, respectively. θ_c is the angle of scattering in the center-of-mass system; θ_{lab} and θ_1 and θ_2 are defined in the diagram of the target geometry in Fig. 2.

The ϵ_{eff} for protons in D_2O ice was determined by measuring the $N_{\max}/Q$ for protons scattered from the oxygen in a thick D_2O ice target. The ice was condensed on a copper target cooled with liquid nitrogen by letting a jet of water vapor containing 99.8 percent

* Assisted by the joint program of the ONR and AEC.
[1] Previous measurements of D_2O are summarized by A. P. French and F. G. P. Seidl, Phil. Mag. **42**, 537 (1951).
[2] Snyder, Rubin, Fowler, and Lauritsen, Rev. Sci. Instr. **21**, 852 (1950).

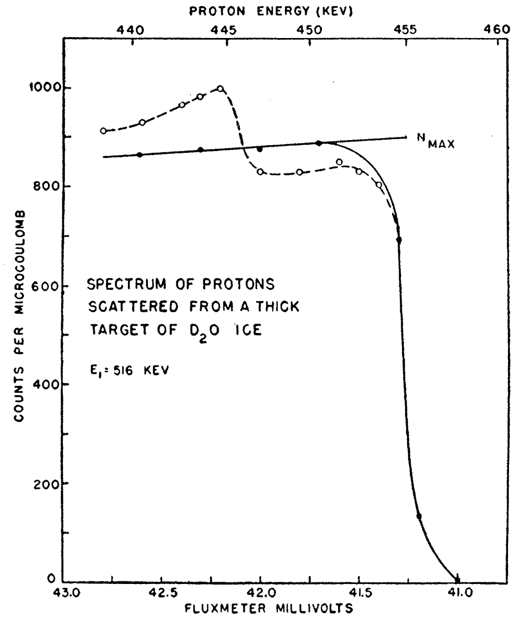

FIG. 1. The spectrum of protons scattered from a thick D_2O ice target. The open points show results with a target which had become contaminated with carbon from residual pump oil vapor in the vacuum system. Solid points were taken with a clean target. $E_1=516$ kev. The spectrometer window ΔE is given by $E/\Delta E=800$.

deuterium fall on the Cu surface. The energy of the incident protons, accelerated in the 600-kev Van de Graaff generator, was measured to within ± 0.5 percent with a 90° electrostatic analyzer. The electrostatic analyzer was calibrated against the $F^{19}(p,\alpha\gamma)O^{16}$ resonance at 340.4 ± 0.4 kev[3] and the linearity of the analyzer was checked by measuring the energy of deuterons scattered from the front face of a carbon target using D^+, DD^+, and DDD^+, beams of 172, 344, and 516 kev, respectively; the energy of the scattered deuterons was constant to within 0.3 percent. The magnetic analyzer was placed to observe particles emitted at an angle of 90.3° with respect to the incident beam, and the target was set at an angle of 45° to make nearly equal angles with incoming and scattered particles.

The scattered proton momentum spectrum is shown in Fig. 1 for $E_1=516$ kev. $N_{\max}$ is obtained by extrapolating the slight rise on the top of the step to the energy E_2 at the midpoint of the step. Also shown in Fig. 1 by the open points are results obtained with an ice surface which had become contaminated with a layer of carbon. Thus the condition of the ice layer, which was too thin to be seen, could be monitored continuously.

To calculate ϵ_{eff} from $N_{\max}$ one must know $(d\sigma/d\omega)$ for protons scattered from oxygen. We have taken for this quantity the value given by the Rutherford formula. Possible nuclear contributions to the cross section are considered in Appendix A, where it is shown that $\sigma_{(\text{nucl.}+\text{Ruth})}/\sigma_{(\text{Ruth})}\approx 1.015$ at 600 kev and the ratio approaches unity for lower energies. Further evidence that the scattering cross section should be given correctly by the Rutherford formula is to be found in the ratios of the $N_{\max}$ values for the protons scattered from the O and Si in a quartz target. This ratio, which is independent of $\epsilon(SiO_2)$, has been measured at five energies between 360 and 550 kev. At each energy the ratio follows the Z^2 dependence within 6 percent, and the average value of this ratio for the five determinations agrees with the theoretical value within 0.3 percent. The 6 percent deviation is within the limits of statistical accuracy of the measurement. Thus if the scattering from oxygen deviates from the Rutherford formula, our measurements indicate that the scattering from Si deviates in the same way.

The value of Ω appearing in the expression for ϵ_{eff} has been measured by placing in front of the spectrometer an aperture subtending a known solid angle at the target. By comparing the yield for this known solid angle with the yield with the aperture removed, the full solid angle of the spectrometer was found to be 0.00490 ± 0.00005 steradian.

The resolution of the spectrometer has been calculated from the geometry of the magnetic field. It has also been measured experimentally by replacing the usual collecting slit with two narrow parallel slits. A spectrum observed with the two slits is the superposition of the two individual spectra, and the displacement between the two spectra is a measure of the resolution of the spectrometer for a collecting slit of width equal to the separation of the two slits. The experimental and theoretical values of R_c agree within 2 percent. Some uncertainty in the calculated value is introduced by the fringing field, and we have used the experimental value of R_c, assigning a probable error of 2 percent.

ϵ_{eff} is a weighted average value of ϵ for incident and scattered particles, which have different energies, and to determine the value of ϵ at a particular energy from the average ϵ_{eff}, one must know the way in which ϵ varies with energy. For protons scattered from oxygen,

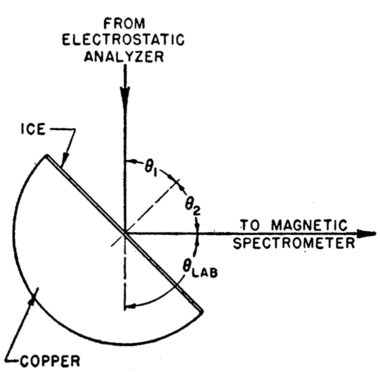

FIG. 2. Schematic diagram of the ice target geometry. The copper target backing is soldered to a liquid nitrogen reservoir, not shown in the diagram.

[3] A. H. Morrish, Phys. Rev. **76**, 1651 (1949).

$E_2=0.88E_1$, so that the difference between E_1 and E_2 is small, and over this small interval a linear variation $\epsilon=A+BE$, where A and B are constants, is a good approximation for proton energies above 200 kev. From this assumption of the form of $\epsilon(E)$, it follows that for $\cos\theta_1=\cos\theta_2$,

$$\epsilon_{\text{eff}}(\partial E_2/\partial E_1+1)^{-1}=\epsilon(E),$$

where

$$E=(E_1\partial E_2/\partial E_1+E_2)(\partial E_2/\partial E_1+1)^{-1}.$$

Thus at bombarding energies of 578, 516, 413, 310, and 258 kev, we have determined the value of ϵ for $E=542$, 484, 387, 291, and 242 kev. These values are plotted in Fig. 3 with the solid points.

III. RELATIVE MEASUREMENTS

The method of measuring ϵ described above was not satisfactory for protons of energy less than 200 kev. Some of the low energy protons capture an electron in the target and emerge as neutral H atoms which would not be deflected in the magnetic field; the amount of this charge neutralization is difficult to measure accurately. A further difficulty at very low energies is the reduced efficiency of our scintillation counter, a thin layer of ZnS on the face of an RCA 5819 photomultiplier tube.

The following method for measuring relative values of ϵ depends only on the measurement of energy ratios and is independent of counter efficiency and neutralization of incident or scattered particles. We first determine accurately the energy E_2 of protons scattered from the surface of a clean Cu target by locating the midpoint of the step in the spectrum. When a thin layer of ice is then formed on the Cu surface, this step is displaced to a lower energy, and to return the step to its original position the bombarding energy must be increased by an amount ΔE_1. If now ΔE_1 is measured for two proton energies, using the same ice layer, then $\Delta E_1'/\Delta E_1=\epsilon_{\text{eff}}'/\epsilon_{\text{eff}}$, and the relative values of ϵ_{eff} are thereby determined. Deuterons can be used as the incident particle, or intercomparison between deuterons and protons can be made. Rapid comparison over a very wide range of proton energy was made possible by accelerating a mixture of D and H ions in the Van de Graaff generator. Using the H^+ beam component, measurements were made of ΔE_1 at the energy E_1 of the generator. Then simply by changing the electrostatic analyzer to pass the DD^+ and HHD^+ ions, measurements were made of ΔE_1 for the deuterons of energy $E_1/2$, which is the same as that for protons of energy $E_1/4$, since it has been shown theoretically and confirmed experimentally that the stopping cross sections for protons and deuterons of the same velocity are equal.[4] Time consuming changes of the generator voltage were thereby avoided.

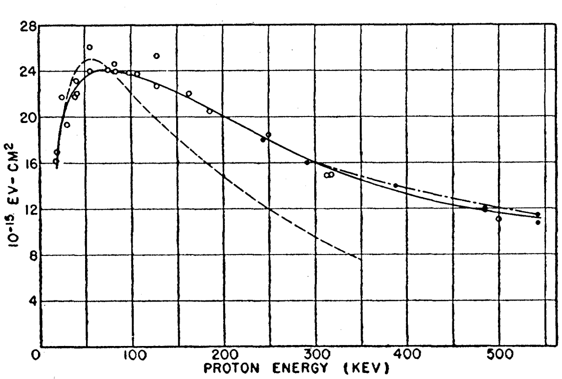

FIG. 3. The proton stopping cross section per D_2O ice molecule. The solid points are absolute measurements discussed in Sec. II. The open circles are relative values discussed in Sec. III. The dashed curve is a summary of previous measurements given in reference 1. The dot-dashed curve shows the theoretical value of $2\epsilon(H)+\epsilon(O)$ taken from reference 6.

TABLE I. Sources of the experimental error, and percentage error introduced in final value of ϵ.

	(%)
Beam integrator	2
Neutralization of incident and scattered protons	<1
Spectrometer resolution	2
Spectrometer solid angle	1
Scattering cross section	<1.5
Counter efficiency	2
Probable error in ϵ:	4

This method requires that the thickness of the ice layer, usually about 3 kev, remain constant while the displacement of the step is located for two different energies, and the measurements must be made as rapidly as possible since the cold target surface will collect residual water and oil vapors present in the vacuum system. ΔE_1 was measured first at one energy and then at the other, and the two measurements were repeated alternately four or five times in succession to establish their ratio; any condensation of foreign material on the target could be detected by a change in the value of ΔE_1. It was necessary to surround the target with a brass cylinder, 2.5 inches in diameter and 1.5 inches high, which was maintained at liquid nitrogen temperature in order to keep the target thickness from increasing through the condensation of residual vapors. The target was thus completely surrounded by the cold surface except for small entrance and exit ports. The vacuum measured outside this cold cylinder varied between 4 and 7×10^{-6} mm of Hg.

These relative measurements of ϵ_{eff} were corrected to give the value of ϵ at a particular energy in the manner described above. This function $\epsilon(E)$ was then normalized to fit the absolute values above 200 kev. The experimental points are shown in Fig. 3 by the open circles.

IV. ACCURACY

The 4 percent probable error of these absolute measurements arises from the uncertainty in the experimental quantities listed in Table I. The incoming beam

[4] C. M. Crenshaw, Phys. Rev. **62**, 54 (1942).

TABLE II. The molecular stopping power $\epsilon = N^{-1}dE/dX$ for protons in D_2O ice.

E_p(kev)	$\epsilon(10^{-15}$ ev-cm$^2)$
18	15.6
20	17.4
30	20.4
40	22.6
50	23.5
60	24.0
70	24.1
80	24.0
100	23.7
125	23.0
150	22.2
200	20.1
300	16.0
400	13.3
500	11.6
540	11.2

was measured with a conventional current integrator calibrated by feeding a known current from a battery and resistance stack to the target. The calibration was checked periodically and is believed to be accurate within 2 percent. The target assembly was held at 300 volts positive with respect to the surrounding chamber to prevent secondary electrons from leaving the target, and the incoming proton beam had to pass through an electric field which would deflect away any electrons in the primary beam. At pressures between 10^{-6} and 10^{-5} mm Hg the neutral component of the incoming beam is negligible for protons above 200 kev. A correction for the small neutral component in the scattered beam has been made using Hall's[5] measurements of the electron capture to loss ratio in several metals. This correction to ϵ is only 3 percent at 240 kev, the lowest energy at which absolute measurements were made.

The efficiency of the ZnS(Ag) scintillation phosphor was measured to be 0.94 ± 0.02 by comparison with a KI(Tl) phosphor, which is assumed to be 100 percent efficient. For both types of phosphor the pulse-height spectrum showed a well-resolved peak, indicating that all of the particles striking the phosphor screen were being counted. The reduced efficiency of the ZnS phosphor is probably due to a transparency of the powdered ZnS screen caused by imperceptible gaps between the individual crystals of the powder.

The most likely source of error in the measurements of the relative values of ϵ is the non-uniformity of the target thickness, such that in changing from one beam component or energy to another, a different thickness of target was bombarded. The beam on the target was restricted to a spot approximately $\frac{1}{16}$ inch in diameter, and variations in the thickness over so small an area do not seem likely. Some uncertainty is introduced in the relative measurements at very low energies by energy straggling in the target. The step in the scattered proton spectrum for the clean Cu target has a very small width ΔE determined by the resolution of the spectrometer, $E/\Delta E = 800$ and the position of the step, taken to be the energy corresponding to half the maximum yield $N_{\max}$, is sharply defined. When the step is displaced by a layer of ice, straggling in the ice rounds off the step and gives it a width amounting to 25 percent of the displacement in the worst cases. If the straggling is truly Gaussian, the energy at half-maximum will still determine the displaced position of the step, but it is not so sharply defined as before. Measurements using the $N_{\max}/3$ energy for the position of the step gave the same relative values. Both the uncertainty due to straggling and possible target non-uniformities would be expected to introduce random deviations rather than systematic error and may account for some of the spread in the experimental points at low energies.

The assumption that the target material is actually D_2O, instead of some other compound of O and D was not checked, but the reproducibility of the experimental results indicates that the composition was constant. The heavy water storage and transfer apparatus was made exclusively of brass, copper, and hard and soft solder; the valve, especially designed for this use, contained no packing, and was not lubricated. The assumption that dE/dX is the same for protons and deuterons of the same velocity has not been tested experimentally[4] to better than 5 percent, but close equality seems reasonable on theoretical grounds.

V. RESULTS

Values of the stopping cross section taken from the smooth curve drawn through the experimental points in Fig. 3 are listed in Table II. Previous measurements of $\epsilon(H_2O)$ are shown by the dashed curve in Fig. 3, taken from reference 1. The agreement is satisfactory below 100 kev, but for higher proton energy our values lie far above the dashed curve, based in this region on Crenshaw's measurements of dE/dX in water vapor.

The dot-dashed curve in Fig. 3 shows the theoretical value for $2\epsilon(H)+\epsilon(O)$ computed by Hirschfelder and Magee[6] from Bethe's semi-empirical theory of stopping power; the empirical constants were evaluated from the range data for natural alpha-particles. The theoretical expression will not apply near the peak of the stopping cross-section curve, and we have not extended the theoretical curve below 300 kev. In the region 300–550 kev, the agreement between the theoretical and experimental values is better than the experimental error. Below 300 kev there is no satisfactory theory. The more detailed treatment of Bethe's theory by Walske[7] will probably hold at somewhat lower energies but unfortunately does not apply simply to light atoms such as oxygen.

One is tempted to conclude from the good agreement between the experimental value of $\epsilon(D_2O)$ (ice) and the

[5] T. Hall, Phys. Rev. **79**, 504 (1950).

[6] J. O. Hirschfelder and J. L. Magee, Phys. Rev. **73**, 207 (1948).

[7] M. C. Walske, Jr., Ph.D. thesis, Cornell University (1951).

theoretical values of $2\epsilon(\mathrm{H})+\epsilon(\mathrm{O})$ that Bragg's law for the addition of stopping cross sections holds very well for water. However, there have been no accurate experimental checks on these theoretical values of $\epsilon(\mathrm{H}_2)$ and $\epsilon(\mathrm{O}_2)$. In addition, there is some experimental evidence[4,8,9] that $\epsilon(\mathrm{H}_2)+\frac{1}{2}\epsilon(\mathrm{O}_2)$, $\epsilon(\mathrm{H_2O})$(vapor), and $\epsilon(\mathrm{H_2O})$(liquid) differ among themselves by more than 10 percent. There have been no previous measurements of $\epsilon(\mathrm{H_2O})$(ice).

We are indebted to Dr. A. P. French of Pembroke College, Cambridge, for valuable discussions of the earlier work on this problem, and to Dr. R. G. Thomas, of this laboratory, for advice on the calculations in Appendix A.

APPENDIX A

Deviations of the scattering cross section $O^{16}(p,p)O^{16}$ from the Rutherford formula are to be expected for two reasons:

(1) *Atomic Electron Cloud.*—The variation in the potential energy of the scattered proton over its wavelength due to the electron cloud is negligible compared with its kinetic energy. Hence we can expect that a calculation of the perturbation of the classical trajectory will lead to a good estimate of the correction to the cross section due to the potential of the atomic electron cloud in the high energy region where neutralization can be neglected.

The angle between asymptotes of the classical motion of the scattered particle is given by

$$\frac{\phi}{2}=\int_0^{u_{\max}} du\{p^{-2}[1-V(u)/E]-u^2\}^{-\frac{1}{2}}, \tag{1}$$

where u = the reciprocal of the radial coordinate, E = the energy in the center-of-mass system, p = the impact parameter, $V(u)$ = the potential energy of the system. $u=u_{\max}$ is obtained by setting the radical in the integrand equal to zero. For the potential $V(u)$ one might use the Fermi-Thomas model but calculations would be difficult. Since the shape of the perturbing potential is probably not important, we have used the following simple form:

$$\begin{aligned} V(u)&=Z_1Z_2e^2u-\Delta \quad \text{for} \quad u>1/a, \\ V(u)&=0 \quad \text{for} \quad u<1/a, \end{aligned}$$

where Δ = the absolute value of the potential at the nucleus due to the electron cloud, $a=Z_1Z_2e^2/\Delta$; Z_1 and Z_2 are the atomic numbers of incident and target particles, respectively, and e is the electronic charge. Foldy[10] gives $\Delta=32.6(Z_2)^{7/5}$ ev, based on the Hartree model of the atom. For scattered protons this gives $a=0.833a_0/(Z_2)^{2/5}$, where a_0 is the Bohr radius. This is fairly consistent with the value for an effective radius of the Fermi-Thomas atom, $a=0.885a_0/(Z_2)^{1/3}$. Equation (1) may now be integrated directly. Expanded in powers of p/a and Δ/E, this gives

$$\phi/2=\phi_0/2+(\Delta/4E)\sin\phi_0, \tag{2}$$

where ϕ_0 is the value of ϕ in Eq. (1) given by setting $\Delta=0$. We note that the correction term is independent of p/a to first order. This tends to confirm the assumption that the shape of the potential due to the electronic cloud is not important. Hence

$$\theta-\theta_0=\delta\theta=-(\Delta/2E)\sin\theta_0, \tag{3}$$

where $\theta=\pi-\phi$ is the scattering angle. To find the correction to the scattering cross section we use the usual relation $(d\sigma(\theta)/d\Omega)d\Omega=2\pi p dp$, where $d\Omega$ is the differential solid angle at the angle θ, and $d\sigma(\theta)$ is the differential scattering cross section at the angle θ.

[8] R. K. Appleyard, Proc. Cambridge Phil. Soc. **47**, 443 (1951).
[9] K. Philipp, Z. Physik **17**, 23 (1923).
[10] L. L. Foldy, Phys. Rev. **83**, 397 (1951).

Since $2\pi p dp=2\pi\sin\theta_0 d\theta_0[d\sigma_R(\theta)/d\Omega]$, where $d\sigma_R$ is the Rutherford cross section, we obtain

$$\left\{\frac{d\sigma(\theta)/d\Omega-d\sigma_R(\theta)/d\Omega}{d\sigma_R(\theta)/d\Omega}\right\}_{\theta_0} =-\left\{d(\delta\theta)/d\theta+\delta\theta\left[\cot\theta+\frac{d(d\sigma_R(\theta)/d\Omega)/d\theta}{d\sigma_R(\theta)/d\Omega}\right]\right\}_{\theta_0}. \tag{4}$$

Substituting Eq. (3) in Eq. (4) gives

$$\left\{\frac{d\sigma(\theta)/d\Omega-d\sigma_R(\theta)/d\Omega}{d\sigma_R(\theta)/d\Omega}\right\}_{\theta_0}=-\frac{\Delta}{E}. \tag{5}$$

For $O^{16}(p,p)O^{16}$ scattering at 200 kev this correction is only −0.3 percent, and has been neglected.

(2) *Nuclear Interference.*—The scattering cross section for a spin $\frac{1}{2}$ particle incident upon a spin 0 nucleus is given by[11]

$$\begin{aligned}\frac{d\sigma(\theta)}{d\Omega}=\frac{1}{k^2}\Bigg\{\Bigg|&-\frac{\eta}{2}\csc^2\frac{\theta}{2}\exp\left(i\eta\ln\csc^2\frac{\theta}{2}\right) \\ &+\sum_{l=0}^{\infty}P_l(\cos\theta)\exp(i\alpha_l)[(l+1)\sin\delta_l^+\exp(i\delta_l^+) \\ &+l\sin\delta_l^-\exp(i\delta_l^-)]\Bigg|^2+\sin^2\theta\Bigg|\sum_{l=1}^{\infty}P_l'(\cos\theta)\exp(i\alpha_l) \\ &\times[\sin\delta_l^+\exp(i\delta_l^+)-\sin\delta_l^-\exp(i\delta_l^-)]\Bigg|^2\Bigg\},\end{aligned}$$

where

$$\begin{aligned} &\eta=Z_1Z_2e^2/\hbar v,\quad k=Mv/\hbar \\ &\exp(i\alpha_l)=(l+i\eta)/(l-i\eta)\cdots(1+i\eta)/(1-i\eta);\quad l>0 \\ &\exp(i\alpha_0)=1 \\ &P_l'(\cos\theta)=dP_l(\cos\theta)/d(\cos\theta) \end{aligned} \tag{6}$$

and v = velocity of relative motion, M = reduced mass of the system, $\delta_l^{\pm}$ are the phase shifts between incident and reflected wave for the lth partial wave, where the total angular momentum of the system is $J=(l\pm\frac{1}{2})$. Because the Coulomb barrier factor will tend to favor the lowest partial wave, and because no resonances involving higher partial waves have been discovered in the energy range to be considered, we shall restrict our attention to interference from S wave nuclear scattering. The effect of a "hard sphere" P wave phase shift has been estimated by Thomas[12] to be negligible.

For S wave interference only, Eq. (7) becomes

$$\frac{d\sigma(\theta)}{d\Omega}=\frac{1}{k^2}\left|-\frac{\eta}{2}\csc^2\frac{\theta}{2}\exp\left(i\eta\ln\csc^2\frac{\theta}{2}\right)+\sin\delta_0\exp(i\delta_0)\right|^2$$

and

$$\cot\delta_0=-\frac{\Theta_0\Phi_0+(f-\Phi_0^*/\Phi_0)^{-1}}{C_0^2x\Phi_0^2} \tag{7}$$

where $\Theta_0\Phi_0$, Φ_0, Φ_0^*/Φ_0 are Coulomb functions tabulated by Bloch *et al.*[13]

$$x=ka,\quad C_0^2=[2\pi\eta/(e^{2\pi\eta}-1)],\quad f=(ka/u)[du/d(kr)],$$

$u=u(r)=r\psi_R(r)$, where $\psi_R(r)$ is the radial part of the wave function. By fitting experimental data on the location of $S_{\frac{1}{2}}$ resonances in F^{17} and its mirror nucleus O^{17} and the value of the $O^{16}(n,n)O^{16}$ scattering cross section at thermal energies, Thomas[12] has evaluated f over the energy range of interest to us, using a nuclear radius $a=5.27\times10^{-13}$ cm. From this information δ_0 and $[d\sigma(\theta)/d\Omega]$ have been determined for bombarding protons of energy 400–600 kev. At a scattering angle of 90° the correction to the Rutherford formula is +1.5 percent at 600 kev ($f=2.52$). Since the effect depends strongly on the barrier factor in this energy region, it is quite small at lower energies and we have neglected it throughout.

[11] R. A. Laubenstein and M. G. W. Laubenstein, Phys. Rev. **84**, 18 (1951).
[12] R. G. Thomas (to be published).
[13] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).